22337

III Semester B.Sc. Examination, November/December 2014
(Semester Scheme)
MATHEMATICS – III

Time : 3 Hours   Max. Marks : 90

Instructions : *1) Answers **all** questions.*
*2) Answers should be written **completely** in **English**.*

I. Answer **any fifteen** of the following. **(15×2=30)**

1) Show that the intersection of any two subgroups of a group G is again a subgroup.

2) Find the order of elements of the group G = { 2, 4, 6, 8} under X_{10}.

3) State Euler's theorem on groups.

4) Find the number of generators of the cyclic group of order 24.

5) Prove that every cyclic group is abelian.

6) Find all the left cosets of H = {0, 3} in the group $(Z_6, +_6)$.

7) Test the convergence of the sequence whose n^{th} term is $\left(\frac{n+1}{n}\right)^{\frac{2n}{n+1}}$.

8) Find the limit of the sequence $\{x_n\} = \left\{\frac{2n^2 + 5\sin\frac{\pi}{n}}{n^2}\right\}$.

9) Verify the sequence $\{x_n\}$ is monotonically increasing or decreasing if $x_n = \left(\frac{2n-7}{3n+2}\right)$.

10) Define convergent series, give an example.

11) Test the convergence of the series $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots \infty$.

P.T.O.

12) If a series Σu_n is convergent then show that $\lim_{n\to\infty} u_n = 0$.

13) Discuss the convergence of $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\ldots$

14) Sum to infinity of the series $1+\frac{x}{3}+\frac{x^2}{5}+\frac{x^3}{7}+\ldots$

15) Show that $\frac{e^x+e^{-x}}{2}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots$

16) Justify the result "Every continuous function need not be differentiable" with an example.

17) Verify Rolle's theorem for the function f(x) = tan x in $[0, \pi]$.

18) Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{\tan 5x}{\tan x}$.

19) Write expression for any two Fourier coefficients of f(x) in $[-\pi, \pi]$.

20) Find the half range sine series for f(x) = x in $[0, \pi]$.

II. Answer **any three** of the following. **(3×5=15)**

1) Prove that a subset H of a group G is a subgroup if and only if $HH^{-1} = H$.

2) In a group G prove that $0(a) = 0(a^{-1})\ \forall, a \in G$.

3) Prove that every subgroup of cyclic group is cyclic.

4) Prove that there exist one-one correspondence between the set of all right cosets and left cosets of a subgroup H of a group G.

5) State and prove Fermat's theorem.

III. Answer **any two** of the following. **(2×5=10)**

1) If $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty} (a_n b_n) = ab$.

2) Discuss the nature of the sequence $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.

3) Examine the behaviour of the sequence

a) $x_n = \frac{(2n^2+3n+4)}{(n+5)} \sin\frac{\pi}{n}$

b) $x_n = \frac{(n+1)^{n+1}}{n^n}$.

IV. Answer **any three** of the following. **(3×5=15)**

1) State and prove nature of p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$.

2) State and prove Raabe's test for positive series.

3) Discuss the nature of the series $\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{x^2}{7.8.9} + \ldots \infty$.

4) Discuss the nature of the series $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \ldots$.

5) Sum to infinity of the series $\frac{1^2}{1!} + \frac{2^2}{2!} + \frac{3^2}{3!} + \ldots$

V. Answer **any two** of the following. **(2×5=10)**

1) A function f(x) defined by $f(x) = \begin{cases} 1+\sin x & \text{for } 0 < x < \frac{\pi}{2} \\ 2+\left(x-\frac{\pi}{2}\right)^2 & \text{for } x \geq \frac{\pi}{2} \end{cases}$.

Examine the differentiability of the function at $x = \frac{\pi}{2}$.

2) State and prove Lagranges mean value theorem.

3) Expand $\sqrt{1+\sin 2x}$ up to term containing x^4 by Maclaurins expansion.

4) Find the value of a and b, so that $\lim_{x \to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$.

VI. Answer **any two** of the following. **(2×5=10)**

1) Obtain the Fourier expansion of the function $f(x) = |x|$ in $[-\pi, \pi]$ and hence deduce $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

2) Obtain the Fourier expansion of the function f(x) defined by $f(x) = \begin{cases} x & \text{in } 0 \le x \le \pi \\ 2\pi - x & \text{in } \pi < x < 2\pi \end{cases}$.

3) Find half range cosine series for $f(x) = e^x$ in $0 < x < L$.

Third Semester B.Sc. Degree Examination, November 2017

(CBCS – Semester Scheme)

Mathematics

Paper 3.1 – REAL ANALYSIS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1) *Answer ALL questions.*
2) *Answers should be written completely in English.*

PART – A

I. Answer any **SIX** questions : **(6 × 2 = 12)**

1. Let $a, b \in R$ and $a \cdot b = 0$, then prove that either $a = 0$ or $b = 0$.

2. Define limit point and isolated point of a set.

3. Find lub and glb of the sequence $\left\{\left(-\frac{1}{2}\right)^n\right\}$.

4. Define Cauchy sequence.

5. Discuss the convergence of the series $1^3 + 2^3 + 3^3 + \cdots + n^3 + \cdots$

6. State Leibnitz's rule for alternating series.

7. State Rolle's theorem.

8. Evaluate $\lim_{x \to 0} \log_x \tan x$.

PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

1. If x and y are real numbers, then prove that $-(x + y) = (-x) + (-y)$.

2. Prove that the intersection of two nbds of a point is also a nbd of that point.

3. Prove that a monotonic increasing sequence bounded above is convergent.

4. Show that the sequence $\{x_n\}$, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ cannot converge by using Cauchy's general principal of convergence.

5. Discuss the convergence of the series $\frac{1}{1\cdot 2\cdot 3} + \frac{3}{2\cdot 3\cdot 4} + \frac{5}{3\cdot 4\cdot 5} + \cdots$.

6. Test the convergence of the series $\frac{1^3}{3} + \frac{2^3}{3^2} + 1 + \frac{4^3}{3^4} + \cdots$.

7. Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in [2, 4].

8. Verify Cauchy's Mean Value theorem for the functions $f(x) = x^3$ and $g(x) = x^2$ in [1, 3].

PART – C

III. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Prove that the set of all rational numbers is countable.

2. If x and y are two real numbers then prove that

   (a) $|x + y| \le |x| + |y|$

   (b) $|x - y| \ge |x| - |y|$.

3. Prove that every non empty set of real numbers which is bounded above has the least upper bound.

4. Prove that a real number x is a limit point of a set S, iff for each $n \in N$, the open interval $\left(x - \frac{1}{n}, x + \frac{1}{n}\right)$ contains a point of S other than x.

IV. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Prove that the limit of a convergent sequence is unique.

2. Test the convergence of the following sequences :

   (a) $\left(1 + \frac{a}{n}\right)^{n/b}$

   (b) $n[\log(n+1) - \log n]$.

3. Show that the sequence $\{x_n\}$, where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2.

4. Show that the sequence $\{x_n\}$ defined by $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{n+n} + \cdots$ is convergent.

V. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Discuss the nature of the Geometric series $\sum_{n=0}^{\infty} x^n$.

2. Discuss the convergence of the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots + \frac{x^n}{n^2+1} + \cdots$.

3. State and prove Raabe's test.

4. Discuss the convergence of alternating series, $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \cdots$.

VI. Answer any **THREE** questions : **(3 × 5 = 15)**

1. Prove that a function which is continuous in a closed interval attains its bounds.

2. State and prove Lagrange's Mean Value theorem.

3. Expand $\cos x$ upto the term containing x^4 by using Maclaurins expansion.

4. Evaluate,

   (a) $\lim_{x \to 0}\left[\frac{1}{x^2} - \frac{1}{x \tan x}\right]$

   (b) $\lim_{x \to 0}\left[\frac{x - \sin x}{x^3}\right]$.

Q.P. Code – 42339

Third Semester B.Sc. Degree Examination, October/November 2019

(CBCS Semester Scheme)

Mathematics

Paper 3.1 – REAL ANALYSIS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answers ALL questions. Answer should be written completely in English.

PART – A

Answer any **SIX** questions : **(6 × 2 = 12)**

1. State rational density theorem.

2. If $a.b = 0$ then prove that either $a = 0$ or $b = 0$.

3. Define convergence and oscillatory sequence.

4. If $\{a_n\}$ is convergent sequence of positive term then evaluate $\lim_{n\to\infty} a_n$ where $a_{n+1} = \frac{6}{5+a_n}$.

5. Write the condition for the convergence and divergence of the geometric series $\sum_{n=0}^{\infty} x^n$.

6. Test the convergence of the series $\sum\left(\frac{n}{n+1}\right)^{n^2}$.

7. State Rolle's theorem for the function $f(x)$.

8. Evaluate $\lim_{x\to 0}\frac{x-\sin x}{x^3}$.

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PART – B

II. Answer any **SIX** questions : **(6 × 3 = 18)**

9. Prove that intersection of two neighbourhoods of a point is also a neighbourhood of that point.

10. Prove that intersection of two open set is an open set.

11. Find the nature of the sequence $a_n = \left(\frac{n-3}{n+2}\right)^{n/3}$.

12. Verify whether the sequence $\{a_n\}$ is monotonic increasing or decreasing where $a_n = \frac{n+3}{n+4}$.

13. Test the convergence of the series $\sum \frac{1}{n}\tan(1/n)$.

14. Discuss the convergence of the series $\sum (-1)^{n-1} \cdot \frac{n}{2n-1}$.

15. Verify Cauchy's mean value theorem for e^x and e^{-x} in $[a,b]$.

16. Expand $\log_e^x$ about $x = 1$ by Taylor's series.

PART – C

III. Answer any **THREE** questions : **(3 × 5 = 15)**

17. Prove that every subset of countable set is countable.

18. Prove that every non empty subset of real numbers is bounded above has the least upper bound.

19. State and prove Archimedean property of real numbers.

20. Let A be a closed set and B be an open set, show that

    (a) $B - A$ is an open set

    (b) $A - B$ is a closed set.

PART – D

IV. Answer any **THREE** questions : **(3 × 5 = 15)**

21. Discuss the behaviour of the sequence whose n th term are

 (a) $n[\log(n+1)-\log n]$

 (b) $\frac{(n+1)^{n+1}}{n^n}$.

22. Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}$ is convergent.

23. Prove that every monotonically increasing sequence and bounded above is convergent.

24. Discuss the nature of the sequence $\{x^n\}$ where x is a real number.

PART – E

V. Answer any **THREE** questions : **(3 × 5 = 15)**

25. Let $\sum u_n$ and $\sum v_n$ be two series of positive terms and $\lim_{n\to\infty}\frac{u_n}{v_n}$ be a finite non zero quantity then prove that $\sum u_n$ and $\sum v_n$ both converge or diverge.

26. Discuss the convergence of the series $\sum \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots 2n}x^n$.

27. State and prove Raabe's test for positive series.

28. Discuss the convergence of the series $\frac{x}{1\cdot 3}+\frac{x^2}{3\cdot 5}+\frac{x^3}{5\cdot 7}+\cdots$

PART – F

VI. Answer any **THREE** questions : **(3 × 5 = 15)**

29. If $f(x)$ is continuous in $[a,b]$ and $f(a) \neq f(b)$, then prove that $f(x)$ takes every value between $f(a)$ and $f(b)$ atleast once.

30. State and prove Lagrange's mean value theorem.

31. Expand $\log(1+x)$ upto term containing x^4 using Maclaurin's series.

32. Evaluate $\lim_{x \to 0} \left[\frac{a^x + b^x}{2}\right]^{\frac{1}{x}}$.

III Semester B.Sc. Examination, November/December 2015
(Semester Scheme)
MATHEMATICS – III

Time : 3 Hours Max. Marks : 90

Instructions : *1) Answer **all** questions.*
*2) Answers should be written completely in **English**.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) If H is a subgroup of a Group G, then prove that $H^{-1} = H$.

2) Find the order of each element of the multiplicative group, $G = \{1, -1, i, -i\}$.

3) Prove that the intersection of two subgroups of a group is also a subgroup.

4) In a cyclic group, G = <a> of order k, and $a^m = a^n$ $(m \neq n)$, then prove that $m \equiv n \pmod{k}$.

5) Find all the Right (left) cosets of $H = \{0, 4, 8\}$ in the Group $(Z_{12}, +_{12})$.

6) Define Index of a subgroup H in a group 'G'.

7) Define Convergence and Divergence of a sequence.

8) Is the sequence $\left\{1+\frac{1}{n}\right\}$ bounded ? If bounded find its bounds.

9) Test the convergence of the sequence $\{n [\log (n+1) - \log n]\}$.

10) If a series $\sum u_n$ is convergent, then prove that $\lim_{n\to\infty} u_n = 0$.

11) Test the convergence of the series $\sum \sin\left(\frac{1}{n}\right)$.

12) Discuss the convergence of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...$

13) Define absolute and conditional convergence of a series.

P.T.O.

14) Find the sum to infinity of the series $\frac{1}{3}+\frac{1}{3.3^3}+\frac{1}{5.3^5}+\ldots$

15) If a function f(x) is differentiable at $x = x_0$, then prove that f(x) is continuous at $x = x_0$.

16) Verify Rolle's theorem for the function $f(x) = x^3 - 4x$ in the interval [–2, 2].

17) Show that, $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ by Maclaurin's expansion.

18) Evaluate, $\lim_{x \to 0} \frac{1-\cos x}{x[\log(1+x)]}$.

19) Find 'a_0' in the Fourier Series expansion of $f(x) = \frac{x^2}{4} + x$ in $[-\pi, \pi]$.

20) Write the expressions for the Fourier co-efficients a_n and b_n in [0, 2L].

II. Answer **any three** of the following : **(3×5=15)**

1) Prove that a subset H of a group G is a subgroup of G if and only if $HH^{-1} = H$.

2) If 'a' is an element of a group G, of order n, then prove that $a^m = e$ for any integer 'm', if and only if n divides m.

3) Prove that every subgroup of a cyclic group is cyclic.

4) Find the number of generators of a cyclic group $(Z_{18}, +_{18})$ and write all its generators.

5) State and prove Euler's theorem in Groups.

III. Answer **any two** of the following : **(2×5=10)**

1) Prove that a monotonic increasing sequence bounded above is convergent.

2) Test the convergence of the sequence whose n^{th} terms are

a) $\left\{\frac{(2n^2+3n+5)}{n+3}\sin\frac{\pi}{n}\right\}$

b) $\{1 + (-1)^n\}$

3) Show that the sequence $\{x_n\}$ defined by $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{n+n}$ converges.

IV. Answer **any three** of the following : **(3×5=15)**

1) Prove that the geometric series $\sum_{0}^{\infty} x^n = 1 + x + x^2 + ...$ is

a) Converges if | x | < |.

b) Diverges if $x \geq 1$.

2) State and prove D′ Alembert's Ratio Test.

3) Discuss the convergence of the series $\frac{x}{1} + \frac{1}{2}.\frac{x^3}{3} + \frac{1.3}{2.4}.\frac{x^5}{5} + \frac{1.3.5}{2.4.6}.\frac{x^7}{7} + ...$

4) Discuss the convergence of the series $\sum \left(\frac{nx}{n+1}\right)^n$.

5) Find sum to infinity of the series $\frac{1}{5} - \frac{1.4}{5.10} + \frac{1.4.7}{5.10.15} - \frac{1.4.7.10}{5.10.15.20} + ...$

V. Answer **any two** of the following : **(2×5=10)**

1) Prove that a function which is continuous in a closed interval attains its bounds.

2) State and prove Lagrange's mean value theorem.

3) Expand the function $\log_e (1 + x)$ upto the term containing x^4 by Maclaurin's expansion.

4) Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$.

VI. Answer **any two** of the following : **(2×5=10)**

1) Obtain the Fourier series for the function $f(x) = x^2$, where $-\pi < x < \pi$ and deduce that, $\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + ...$

2) Find the Fourier Series for the function $f(x) = \begin{cases} -1 & \text{when} \quad -3 < x < 0 \\ 0 & \text{when} \quad x = 0 \\ 1 & \text{when} \quad 0 < x < 3 \end{cases}$.

3) Find the half range sine series for the function f(x) = x when, 0 < x < 2.

III Semester B.Sc. Examination, Nov./Dec. 2016
(Semester Scheme)
Paper – III : MATHEMATICS

Time : 3 Hours   Max. Marks : 90

Instruction : *1) Answer **all** questions.*
*2) Answer should be written **completely** in **English**.*

I. Answer **any fifteen** of the following : **(15×2=30)**

1) Find the order of the each element of the group $(Z_6, +_6)$.

2) Find the number of generator of the cyclic group of order 21.

3) Find the right co-sets of subgroup H = {1, 3, 9} in the group $(Z_{13} - \{0\}, X_{13})$.

4) If 'a' is a generator of a cyclic group 'G' then a^{-1} is also a generator.

5) Find the index of the group of order 12, whose subgroup is of order 3.

6) State Fermat's theorem on groups.

7) Test the convergence of sequence $\left(\frac{n-3}{n+2}\right)^{n/3}$.

8) Show that the sequence $x_n = \frac{n-4}{n-3}$ is bounded.

9) Show that the sequence $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ is monotonically increasing.

10) Test the convergence of the series $\sum \frac{\sqrt{n}}{(2n+3)}$.

11) State Cauchy's n^{th} root test.

12) Test the convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{(3n-1)}$.

13) Test the convergence of the series $\sum\frac{x^n}{n^n}$.

14) Find sum to infinity of the series $\frac{1}{7}+\frac{1}{3.7^3}+\frac{1}{3.7^5}+\ldots$

15) Examine the differentiability of the function $f(x)=\begin{cases}x^2 & \text{if } x\le 3\\ 6x-9 & \text{if } x>3\end{cases}$ at $x=3$.

16) Verify Lagrange's mean value theorem for the function f(x) = logx in [1, e].

17) Obtain the Maclaurin's series expansion for the function coshx up to the terms containing x^4.

18) Evaluate $\lim\limits_{x\to 0}\frac{x-\sin x}{x^3}$.

19) Calculate 'a_0' in the Fourier series expansion of $f(x)=x^2$ in $-\pi<x<\pi$.

20) Obtain half range sine series for f(x) = x in 0 < x < 2.

II. Answer **any three** of the following : **(3×5=15)**

1) If H and K are any two subgroups of a group 'G'. Then prove that, HK is subgroup of G if and only if HK = KH.

2) Let 'G' be a cyclic group of order 'd' and 'a' be the generator. The element $a^k (k < d)$ is also a generator of 'G' if and only if (k, d) = 1.

3) If 'H' is a subgroup of 'G'. Then show that there exist a one to one correspondence between any two right (or left) co-sets of H in G.

4) State and prove Lagrange's theorem on finite groups.

5) If 'n' is any positive integer and 'a' is relatively prime to 'n' then prove that $a^{\phi(n)}\equiv 1(\text{mod } n)$.

III. Answer **any two** of the following : **(2×5=10)**

1) Prove that every monotonic decreasing sequence bounded below is convergent.

2) Test the convergence of the following sequence whose n^{th} terms are,

a) $\left\{\sqrt{n}\left(\sqrt{n+3}-\sqrt{n+2}\right)\right\}$

b) $\left\{\left(\frac{n+1}{n}\right)^{\frac{2n^2}{n+1}}\right\}$

3) Discuss the nature of the sequence $\{x^n\}$, where x is real number, $x \in [0, 1]$.

IV. Answer **any three** of the following : **(3×5=15)**

1) Discuss the nature of the series $\sum \frac{1}{n^p}$.

2) Discuss the convergence of the series $\frac{1}{3}x+\frac{1.2}{3.5}x^2+\frac{1.2.3}{3.5.7}x^3+\ldots$

3) Discuss the convergence of the series $\sum\left(1+\frac{1}{n}\right)^{-n^2}$.

4) State and prove Leibnitz's test for an alternating series.

5) Find sum to infinity of the series $\frac{3}{1}+\frac{3.5}{1.2}\cdot\frac{1}{3}+\frac{3.5.7}{1.2.3}\cdot\frac{1}{3^2}+\ldots$

V. Answer **any two** of the following : **(2×5=10)**

1) Prove that a function which is continuous in a closed interval, takes every value between its bounds atleast once.

2) State and prove Rolle's theorem.

3) Expand log(1 + cosx) upto the term containing x^3, by using Maclaurin's expansion.

4) Evaluate, $\lim_{x\to\pi/2}(\sec x)^{\tan x}$.

VI. Answer **any two** of the following : **(2×5=10)**

1) Find the Fourier series of $f(x) = \begin{cases} x + \frac{\pi}{2} & -\pi < x < 0 \\ \frac{\pi}{2} - x & 0 \le x < \pi \end{cases}$

2) Find the Fourier series of the function $f(x) = x^2 - 2$, in the interval $-2 \le x \le 2$.

3) Find half range sine series for the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$.

Sri Siddharth First Grade College-Tumkur

Q.P. Code – 22337

Third Semester B.Sc. Degree Examination, October/November 2019

(Semester Scheme)

Paper III – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates : Answers ALL questions. Answers should be written completely in English.

I. Answer any **FIFTEEN** of the following : **(15 × 2 = 30)**

1. Define the subgroup of a group.

2. Find the order of each element of the multiplicative group $G=\{1, -1, i, -i\}$.

3. Show that the group $G=\{1, w, w^2\}$ of the cube roots of unity w.r.t. multiplication is a cyclic group.

4. Show that '*1*' is a generator of the group (z_n, t_n).

5. Define the index of a subgroup *H* of a group *G*.

6. Find all the subgroups of the group (Z_{10}, t_{10}).

7. Define infimum and supremum of a sequence.

8. Find the limit of the sequence $\left\{n\sin\left(\frac{1}{n}\right)\right\}$.

9. Test the convergence of the sequence $\{1-(-1)^n\}$.

10. Show that the series $\sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$ is convergent.

11. State Cauchy's root test for convergence of a series of positive terms.

12. Test the nature of the series $\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdots(2n-1)}{2\cdot 4\cdot 6\cdots(2n)}$.

13. Discuss the nature of the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$.

14. Find the sum to infinity of the series $1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\pm\cdots$.

15. If $g(x)=\begin{cases} x-2 & \text{for } x<0 \\ 2x-1 & \text{for } x>0 \end{cases}$, find $\lim_{x\to 0}[g(x)]$, if it exists.

16. State Lagrange's mean value theorem.

17. Show that $\cosh x = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots$ using Maclaurin's series.

18. Using L'Hospital's rule, evaluate $\lim_{x\to 0}[x\log(\tan x)]$.

19. Write the Fourier expansion of an even function $f(x)$ defined in the interval $(-l,l)$.

20. Obtain the half range sine series expansion of $f(x)=x^2$ in (0, 2)

II. Answer any **THREE** of the following. **(3 × 5 = 15)**

21. Prove that a subset H of a group G is a subgroup of G if and only if $HH^{-1}=H$.

22. If 'a' is any element of a group of G, of order 'n', then prove that $a^m=e$ for any integer 'm' if and only if n divides m.

23. Define a cyclic group. Prove that every cyclic group is Abelian.

24. Prove that there exists a one-to-one correspondence between two right cosets of a subgroup H of a group G.

25. Prove that, if 'a' is any integer and 'p' is any positive prime, then $a^p \equiv a(\bmod p)$.

III. Answer any **TWO** of the following. **(2 × 5 = 10)**

26. If $\{a_n\}$ and $\{b_n\}$ are sequences such that $\lim_{n\to\infty} a_n = l$ and $\lim_{n\to\infty} b_n = m$, then prove that $\lim_{n\to\infty}(a_n \pm b_n) = l \pm m$.

27. Discuss the nature of the sequence $\{x^n\}$, $x \in \Re$.

28. Show that the sequence $\{x_n\}$, where $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots \frac{1}{n+n}$, is convergent.

IV. Answer any **THREE** of the following. **(3 × 5 = 15)**

29. Prove that, if a series $\sum_{n=1}^{\infty} u_n$ is convergent, then $\lim_{n\to\infty} u_n = 0$. Further, give an example to prove that the converse is not true.

30. State and prove D'Alembert's ratio test for convergence of a series of positive terms.

31. Test the nature of the series $\sum_{n=1}^{\infty}\left(\frac{nx}{n+1}\right)^n$.

32. Discuss the convergence of $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{2n-1}$.

33. Sum of infinity the series $\frac{3\cdot5}{3\cdot6} + \frac{3\cdot5\cdot7}{3\cdot6\cdot9} + \frac{3\cdot5\cdot7\cdot9}{3\cdot6\cdot9\cdot12} + \cdots$

V. Answer any **TWO** of the following : **(2 × 5 = 10)**

34. Show that $f(x) = \begin{cases} x^2 - 1 & \text{if} \quad x > 1 \\ 1 - x & \text{if} \quad x < 1 \end{cases}$ is not differentiable at $x = 1$.

35. State and prove Rolle's theorem.

36. Evaluate $\lim_{x\to 0}\left[\frac{1}{x^2} - \cot^2 x\right]$ using L'Hospital's rule.

37. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{1/x}$.

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VI. Answer any **TWO** of the following : **(2 × 5 = 10)**

38. Obtain the Fourier series expansion of $f(x)=\begin{cases}-k & \text{if} \quad -\pi \le x < 0 \\ k & \text{if} \quad 0 \le x \le \pi\end{cases}$.

39. Expand $f(x)=x-x^2$ as a Fourier series in (-1, 1).

40. Obtain the half range cosine series expansion of $f(x)=\begin{cases}x & \text{if} \quad 0 \le x < \pi \\ 2\pi - x & \text{if} \quad \pi < x \le 2\pi\end{cases}$.
